Solve for x, y, z

$$\begin{cases} 1x - y + 2z = 15 \\ 1y + 2z = 8 \\ 1z = 5 \end{cases}$$

Row echelon form

back substitution

$(3, -2, 5)$

Gaussian Elimination

1. Interchange two rows
2. Add or subtract any two rows
3. Multiply by any constant

Goal → row echelon form

$$\begin{cases} 1\ \#\ \#\ =\ \# \\ \quad 1\ \#\ =\ \# \\ \quad\quad 1\ =\ \# \end{cases}$$

$$\begin{cases} 2x - 4y + 6z = -6 \\ -x + 2y + 4z = 17 \\ x + 2y - 2z = 3 \end{cases} \Rightarrow \begin{cases} x + 2y - 2z = 3 \\ -x + 2y + 4z = 17 \\ 2x - 4y + 6z = -6 \end{cases}$$

switch r_1 + r_3

(-2) $-2x - 4y + 4z = -6$

$$\begin{cases} x + 2y - 2z = 3 \\ 4y + 2z = 20 \quad r_1 + r_2 \\ 2x - 4y + 6z = -6 \end{cases} \Rightarrow \begin{cases} x + 2y - 2z = 3 \\ 8y + 4z = 40 \\ 4y + 2z = 20 \quad -2r_1 + r_3 \\ -8y + 10z = -12 \end{cases}$$

$$\begin{cases} x + 2y - 2z = 3 \\ 4y + 2z = 20 \quad 2r_2 + r_3 \\ 14z = 28 \end{cases} \Rightarrow \begin{cases} x + 2y - 2z = 3 \\ y + \frac{1}{2}z = 5 \quad \frac{1}{4}r_2 \\ z = 2 \quad \frac{1}{14}r_3 \end{cases}$$

$(-1, 4, 2)$

$$\begin{cases} 2x + 2y - z = 10 \\ x - 2y + z = -4 \\ -4x + y - 2z = 1 \end{cases} \Rightarrow$$

$-2x + 4y - 2z = 8$

$x - 2y + z = -4$ ✓

$2x + 2y - z = 10$ switch $R_1 + R_2$

$-4x + y - 2z = 1$

$4x - 8y + 4z = -16$

$$\begin{cases} x - 2y + z = -4 \\ 6y - 3z = 18 \\ -4x + y - 2z = 1 \end{cases} \quad -2r_1 + r_2$$

$$\begin{cases} x - 2y + z = -4 \\ 6y - 3z = 18 \\ -7y + 2z = -15 \end{cases} \quad 4r_1 + r_3$$

$7x - 3.5z = 21$

$$\begin{cases} x - 2y + z = -4 \\ y - \frac{1}{2}z = 3 \\ -7y + 2z = -15 \end{cases} \quad \frac{1}{6}r_2$$

$$\begin{cases} x - 2y + z = -4 \\ y - \frac{1}{2}z = 3 \\ -15z = 6 \end{cases} \quad 7r_2 + r_3$$

$$\begin{cases} x - 2y + z = -4 \\ y - \frac{1}{2}z = 3 \\ z = -4 \end{cases} \quad (2, 1, -4)$$

7.3

#2, 9, 13-16, 37, 40, 77, 78